Int. J. Solids Structures Vol. 16, pp. 61–70
Pergamon Press Ltd., 1980. Printed in Great Britain

VIBRATION AND BUCKLING OF PLATES AT ELEVATED TEMPERATURES

R. JONES† and J. MAZUMDAR
Department of Applied Mathematics, The University of Adelaide, South Australia

and

Y. K. CHEUNG‡
Department of Civil Engineering, The University of Adelaide, South Australia

(*Received 7 July 1978; in revised form 22 January 1979*)

Abstract—The linear and non-linear dynamic behavior of plates at elevated temperatures are examined. Analytical solutions for the buckling and post-buckling behavior are obtained. A general formula is then presented which links the fundamental frequency of vibration to the critical buckling temperature and the corresponding frequency of the unheated plate. The behavior of certain visco-elastic plates is also considered and a criterion for thermal buckling is presented. The analysis may be interpreted as an extension of Williams analysis of long narrow plates, for plates of finite aspect ratio.

1. INTRODUCTION

A simple and yet sufficiently accurate method for the analysis of static and dynamic behavior of heated plates has recently been suggested by Pal[1–3]. This method is based upon the "Berger technique"[4] which neglects the second strain invariant in the expression for the total strain energy of the plate. The governing equations thus obtained are uncoupled quasi linear differential equations. Pal has established that Berger's approach yields results sufficient for all practical purposes in engineering.

The purpose of the present investigation is to study the vibration of both elastic and viscoelastic plates at elevated temperatures using the Berger technique in greater detail and obtain a general formula which links the fundamental frequency to the critical buckling temperature and the corresponding fundamental frequency of the unheated plate. A criterion for thermal buckling is then presented. On the basis of this analysis a modification of Pal's work is then proposed which may be used when the Berger technique is not sufficiently accurate.

2. ANALYSIS OF HEATED PLATES

By neglecting the second strain invariant and using Hamilton's principle, Pal[2] obtained the following set of approximate equations of motion:

$$D\nabla^4 w + k^2\nabla^2 w = q(x,y,t) - \rho h\frac{\partial^2 w}{\partial t^2} - \nabla^2\frac{M_T}{1-\nu} \tag{1}$$

$$\frac{N_T}{1-\nu} - \frac{12D}{h^2}e_1 = k^2 \text{ (const.)} \tag{2}$$

where e_1 is the first strain invariant given by

$$e_1 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{2}|\underset{\sim}{\nabla} w|^2. \tag{3}$$

Here u, v and w are the displacement components in the median surface in x, y and z directions respectively, ρ and ν are the density and Poisson's ratio of the plate material, h is the plate

†Present address: Aeronautical Research Laboratories, Melbourne, Australia.
‡Present address: Department of Civil Engineering, University of Hong Kong.

thickness, D is the flexural rigidity, $q(x,y,t)$ is the normal pressure and, N_T and M_T are given by

$$N_T = \int_{-h/2}^{h/2} E\alpha T \, dz, \tag{4}$$

and

$$M_T = \int_{-h/2}^{h/2} E\alpha z T \, dz \tag{5}$$

α being the coefficient of linear thermal expansion of the plate material, E is the Young's modulus and T is the applied temperature field.

The corresponding equation for the special case which gives rise to a thermal buckling problem is obtained by putting $q = \partial^2 w/\partial t^2 = 0$ and $M_T = 0$. In order to assess the accuracy of the Berger technique for this case we will compare the predicted values of the critical temperature with those values obtained by more classical methods of analysis.

In this case eqn (1) becomes

$$D\nabla^4 w + k^2\nabla^2 w = 0 \tag{6}$$

which is identical to the governing equation for the mechanical buckling of a plate subjected to uniform compression $N_x = N_y(= N)$, $N_{xy} = 0$. Consequently, in order for a non-trivial solution to occur we require

$$k^2 = N_c \tag{7}$$

where N_c is the critical buckling load. The solution for w is thus taken to be

$$w = \delta w_b(x,y) \tag{8}$$

where δ is an arbitrary parameter and w_b is the deformation due to biaxial loading, and satisfies

$$\nabla^4 w_b + \frac{N_c}{D}\nabla^2 w_b = 0. \tag{9}$$

If we non-dimensionalize w_b such that its maximum value is unity, then δ may be interpreted as the maximum deflection of the plate and has the dimension of length.

The temperature deflection curve is now obtained by substituting for w in eqn (2), viz.

$$N_c = \frac{N_T}{1-\nu} - \frac{12D}{h^2}\left(\frac{\partial u}{\partial x} + \frac{\partial y}{\partial y} + \frac{\delta^2}{2}|\underline{\nabla} w_b|^2\right) \tag{10}$$

which in the case of a plate whose edges are restrained from horizontal motion gives

$$\frac{\delta^2}{h^2} = \frac{\int\int\left(\frac{N_T}{1-\nu} - N_c\right) dx\, dy}{6D\int\int |\underline{\nabla} w_b|^2 \, dx\, dy} \tag{11}$$

where the integrations are carried over the total area of the plate. We thus see that the critical buckling temperature occurs when

$$\int\int\left(\frac{N_T}{1-\nu} - N_c\right) dx\, dy = 0. \tag{12}$$

Consequently, eqn (11) may be written as

$$\frac{\delta^2}{h^2} = \frac{\int\int (N_T - N_{Tc})\,\mathrm{d}x\,\mathrm{d}y}{(1-\nu)6D\int\int |\nabla w_b|^2\,\mathrm{d}x\,\mathrm{d}y} \tag{13}$$

where we have denoted the value of N_T corresponding to the critical temperature T_c as N_{Tc}.

The above analysis is quite general and holds for plates of any shape and support conditions. Changing either the geometry or the support conditions changes the value of N_c and the deflected shape, i.e. $w_b(x,y)$. Because of the similarity in the governing equations the deflection at any point in the plate may be obtained using standard plate buckling routines, or known solutions.

In order to assess the accuracy of the method we will consider the following cases.

Case (1). A rectangular plate with sides of length $2a$ and $2b$ subject to the parabolic temperature variation

$$T(x,y,z) = T_0 + T_1\left(1-\left(\frac{x-a}{a}\right)^2\right)\left(1-\left(\frac{y-b}{b}\right)^2\right) \tag{14}$$

where T_0 and T_1 are constants and where the edges of the plate are simply supported.

In this case, we have [5]

$$w_b = \sin\frac{\pi x}{2a}\sin\frac{\pi y}{2b} \tag{15}$$

while the values of T_c are taken from Ref. [6]. The temperature-deflection relationship is immediately obtained from eqns (4) and (13) and is shown graphically for aspect ratios 1 and 3 with T_0/T_1 as a parameter along with the corresponding results given in [6] (see Fig. 1). It is seen that there is excellent agreement between the results obtained.

Case (2). A circular plate of radius b subjected to a uniform temperature field $T(x,y,z) = T$. In this case [5],

$$\frac{\mathrm{d}w_b}{\mathrm{d}r} = \frac{\gamma J_1(\gamma r)}{J_0(\gamma b)} \tag{16}$$

where $\gamma b = 2.05$ for simply supported edges and 3.83 for clamped edges. Here, J_0 and J_1 are Bessel functions of the order zero and one respectively. The value of N_{T_c} is evaluated using eqn (12) and is found to be

$$N_{T_c} = \begin{cases}(1-\nu)(2.05)^2hD/b^2, & \text{for simply supported edges}\\ (1-\nu)(3.83)^2hD/b^2, & \text{for clamped edges.}\end{cases} \tag{17}$$

Figure 2 shows the resultant relationship for both a clamped and a simply supported plate along with the results given in [7].

Case (3). A long narrow plate strip of width b subjected to an arbitrary temperature variation.

In this case assuming the x-axis in the direction of the longest side, the component of the deflection in the y direction, i.e. v, is neglected in comparison with the components in the other two directions, i.e. u and w, which are now regarded as functions of x alone. Consequently, eqns (6) and (2) now become respectively,

$$D\frac{\mathrm{d}^4w}{\mathrm{d}x^4} + k^2\frac{\mathrm{d}^2w}{\mathrm{d}x^2} = 0 \tag{18}$$

and

$$k^2 = \frac{N_T}{1-\nu} - \frac{12D}{h^2}\left(\frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2 + \frac{\mathrm{d}u}{\mathrm{d}x}\right) \tag{19}$$

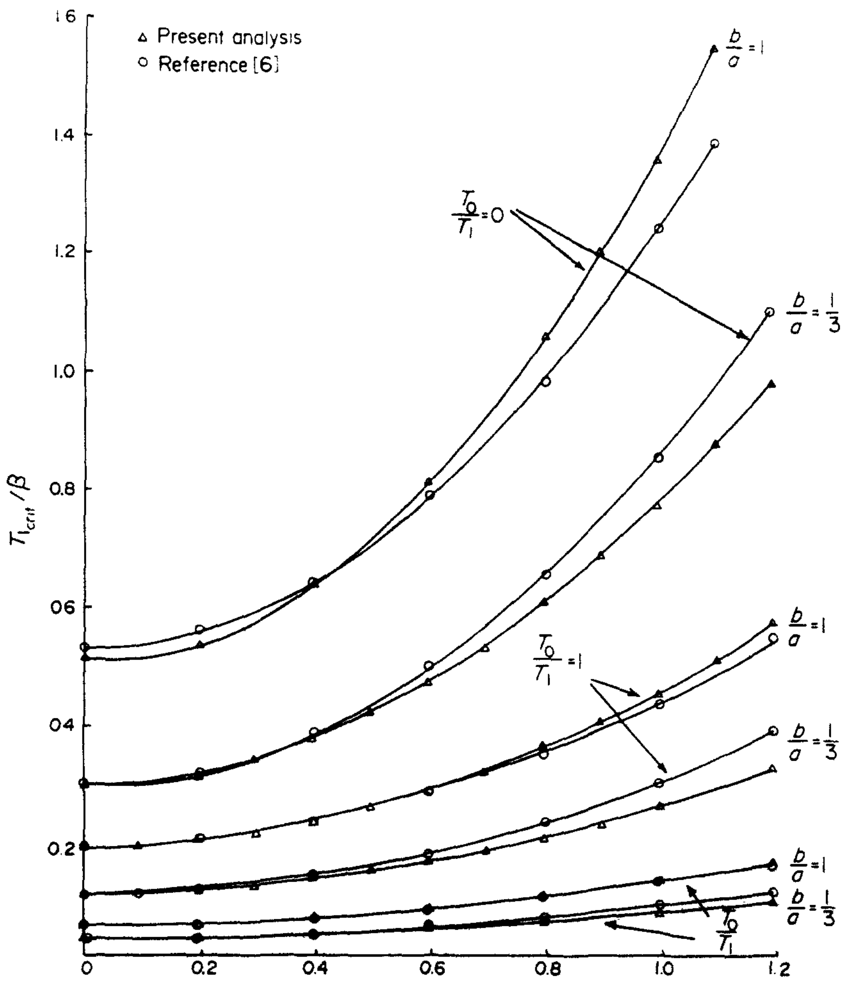

Fig. 1. Comparison of results for central deflection vs temperature rise: $b/a = 1.0$, $1/3$, $\beta = 1/(\alpha(1-\nu^2))(h/b)^2$, α = coefficient of linear thermal expansion.

which coincide exactly with the governing equations for a plate strip as derived by Williams [8].

For plates of finite aspect ratio the resultant approximation is dependent upon the value of the critical buckling temperature. Further, in accordance with eqn (12) the present method has been shown to predict that

$$\iiint T_c \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \frac{AN_c(1-\nu)}{E\alpha} \tag{20}$$

where A denotes the total area of the plate and where the triple integral is over the volume of the plate. In the case of the uniformly heated clamped circular plate considered above, eqn (20) gives a critical temperature of 14.67 $(1-\nu)Dh/b^2\alpha E$ as against the value of 14.72 $(1-\nu)Dh/b^2\alpha E$ given in [9]. However eqn (20) is less accurate for plates of finite aspect ratio subject to non-uniform temperature fields as can be seen in Fig. 1.

As a result of this investigation we conclude that the Berger technique provides an adequately satisfactory method for examining the buckling behavior of plates at elevated temperatures.

3. VIBRATION OF PLATES

Let us now examine the case of small amplitude vibration of plates at elevated temperatures.

In this case the expression for k^2 is given by

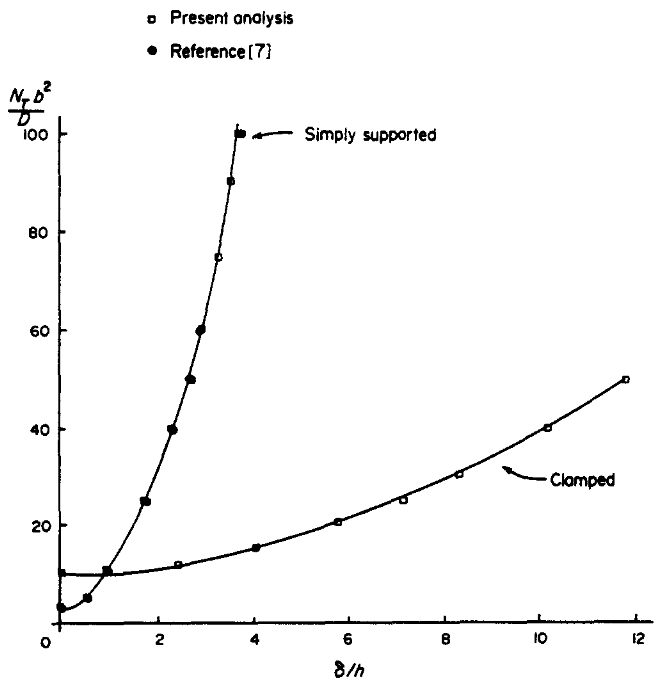

Fig. 2. Buckling behavior of a circular plate.

$$k^2 = \frac{N_T}{1-\nu} - \frac{12D}{h^2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\right) \tag{21}$$

so that if the edges of the plate are restrained from horizontal motion then the value of k^2 is given by

$$k^2 = \frac{1}{A}\int\int \frac{N_T}{1-\nu}\,\mathrm{d}x\,\mathrm{d}y = N_T^*/1-\nu \tag{22}$$

where N_T^* is the mean value of N_T, over the area of the plate. Consequently substituting for k^2 in eqn (1) we obtain

$$D\nabla^4 w + \frac{N_T^*}{1-\nu}\nabla^2 w = q - \rho h\frac{\partial^2 w}{\partial t^2} - \nabla^2\frac{M_T}{1-\nu} \tag{23}$$

which represents the resulting governing equations for the small amplitude dynamic response of heated plates. Although this analysis is based upon an approximate technique, eqn (23) is known to be exact for a polygonal plate subjected to a uniform temperature field[10].

Let us now consider the free vibration problen which arises when $q = M_T = 0$ and for which eqn (23) reduces to

$$D\nabla^4 w + \frac{N_T^*}{1-\nu}\nabla^2 w = -\rho h\frac{\partial^2 w}{\partial t^2}. \tag{24}$$

If the temperature T is time independent then eqn (24) is identical to the governing equation for the vibration of a plate with uniform boundary tension $N_x = N_y = N_T^*/1-\nu$, $N_{xy} = 0$. Consequently when studying the free vibration problem for heated plates we may utilize existing computer programs and known results for vibrating plates with inplane loads. As a result of this analogy we are able to predict that the temperature-frequency relationship for a simply supported polygonal plate is given by

$$\frac{N_T^*}{N_{T_c}^*} + \frac{\omega^2}{\omega_c^2} = 1 \tag{25}$$

where $N_{T_c}^*$ is the mean value of the critical buckling temperatüre, ω is the fundamental frequency of vibration of the heated plate and ω_c is the fundamental frequency of vibration of the unheated plate. This analogy is an extension of the relationship proposed by Lurie[11] and coincides with Lurie's formulae in the case of a uniform temperature field. Furthermore, as a consequence of the analysis presented in [12] eqn (25) also holds for both simply supported and clamped circular and elliptical plates, and as mentioned in [13] may be used as an accurate approximation in the case of a clamped retangular plate.

A further check on the accuracy of this analysis may be obtained by considering the case of a vibrating clamped square plate. This problem has recently been considered in Ref.[14] where the plate was subjected to three distinct temperature fields, two of which were non-uniform. The frequencies obtained in[14] each satisfy eqn (25), the temperature-frequency squared curves being linear in each case. This excellent agreement in the case of a clamped square plate coupled with the fact that the analysis is known to be exact in the case of a simply supported polygonal plate with uniform temperature field and the accuracy of the method for static problems, as seen in Section 2, gives us confidence in claiming that the Berger technique is an accurate method for obtaining close approximate solutions for the vibration of heated plates.

4. VISCOELASTIC ANALYSIS

Let us now consider the small amplitude response of a thermo-rheologically simple viscoelastic plate. In this case in accordance with the well-known correspondence principle we have the governing differential equation, corresponding to eqn (23) as

$$D(p)\nabla^4 w + \frac{N_T^*}{1-\nu(p)}\nabla^2 w = q - p^2 \rho h w - \frac{\nabla^2 M_T}{1-\nu(p)} \tag{26}$$

where p denotes the operator $p \equiv (\partial/\partial t)$, and where $D(p)$ and $\nu(p)$ are the viscoelastic time operators corresponding to the flexural rigidity D and Poisson's ratio ν in the elastic case. Equation (26) is however identical to the governing equation for the analysis of a viscoelastic plate with inplane loads $N_x = N_y = N_T^*/1-\nu(p)$ and transverse load $q - (\nabla^2 M_T)/(1-\nu(p))$. The behaviour of such viscoelastic plates has been studied in detail[15–17] so that as a consequence of this analogy we are able to present some interesting results for the behavior of viscoelastic plates at elevated temperatures. For example, if we consider $q = \nabla^2 M_T = 0$ and inertia effects are negligible, then as in Ref.[16], we may seek the deflection $w(x,y,t)$ as separable in time and space

$$w(x,y,t) = W(x,y)\,\tau(t) \tag{27}$$

where $W(x,y)$ satisfies

$$\nabla^4 W - C\nabla^2 W = 0 \tag{28}$$

and where the function $\tau(t)$, which represents the magnitude of the deflection, satisfies

$$D(p)\tau(t) - \tau(t)\frac{N_T^*}{C(1-\nu(p))} = 0. \tag{29}$$

Here C represents a separation constant. Consequently we see that the "mode shapes" are time independent and that only their amplitudes vary with time. If N_T^* is strictly constant then, in accordance with Ref. [16], there exists an upper and a lower critical temperature. The lower critical temperature corresponds to zero deflection rates while the upper critical temperature corresponds to an infinite deflection. Setting $p = 0$ in eqn (29) determines the lower critical temperature. This gives

$$C = \frac{N_T^*}{(1-\nu(0))D(0)} \tag{30}$$

so that eqn (28) becomes

$$\nabla^4 W - \frac{N_T^*}{D(0)(1-\nu(0))}\nabla^2 W = 0. \tag{31}$$

Similarly the upper critical temperature is found by setting $p=\infty$ in eqn (20) which subsequently gives

$$\nabla^4 W - \frac{N_T^*}{D(\infty)(1-\nu(\infty))}\nabla^2 W = 0 \tag{32}$$

where the notation used is the same as given in Ref.[16]. If the boundary conditions contain Poisson's ratio (as is the case for simply supported or free edges) then when solving eqn (31) or (32) we merely replace ν by $\nu(0)$ or $\nu(\infty)$ respectively. For Kelvin and Maxwell material upper and lower critical buckling loads are expressible in terms of the elastic critical loads. Consequently the present analogy indicates a similar relationship for the upper and lower critical temperatures. This relationship is given in Table 1.

Table 1.

Material	Lower critical temp.	Upper critical temp.
Elastic	Standard elastic critical temp.	
Maxwell	0	Elastic critical temp.
Kelvin	Elastic critical temp.	∞

A similar behavior is obtained for viscoelastic columns. The buckling criteria for viscoelastic columns has been discussed in [18] so that using the analogy discussed above we can immediately present the upper and lower critical temperatures, for various viscoelastic beams. Table 2 presents the upper and lower critical values of N_T^* thus obtained.

Here I and L are the moment of inertia and the length of the beam respectively. In calculating the lower critical temperature the modulii are evaluated using $E_1(p)=E_1(0)$, etc. while for the upper critical temperature we take $E_1(p)=E_1(\infty)$, etc. Here the column considered is simply supported at each end.

The physical interpretation of the critical temperature is the same as for the critical buckling loads, discussed in [16–18]. If $N_T^* <$ its lower critical value the deflection decreases with time. If N_T^* equals its lower critical value then the deflection is constant; if the lower critical value $< N_T^* <$ upper critical value, the deflection increases with time while if N_T^* equals its upper critical value then the deflection is immediately infinite.

Table 2. Critical temperatures for the viscoelastic models illustrated in Fig. 3

Model	Lower critical value of $N_T^*/1-\nu(0)$	Upper critical value of $N_T^*/1-\nu(\infty)$
(a)	0	$\frac{\pi^2 E_1 I}{L^2}$
(b)	$\frac{\pi^2 E_1 I}{L^2}$	∞
(c)	$\frac{\pi^2 E_1 E_2 I}{L^2(E_1+E_2)}$	$\frac{\pi^2 E_2 I}{L^2}$
(d)	0	$\frac{\pi^2 E_2 I}{L^2}$
(e)	0	$\frac{9\pi^2 KGI}{L^2(3K+G)}$
(f)	$\frac{9\pi^2 KGI}{L^2(3K+G)}$	$\frac{9\pi^2 KI}{L^2}$

(Here it should be recalled that N is defined as $N_T = E\alpha \int_{-h/2}^{h/2} T\,dz$ so that the upper and lower critical values of N_T correspond to the upper and lower critical temperatures.)

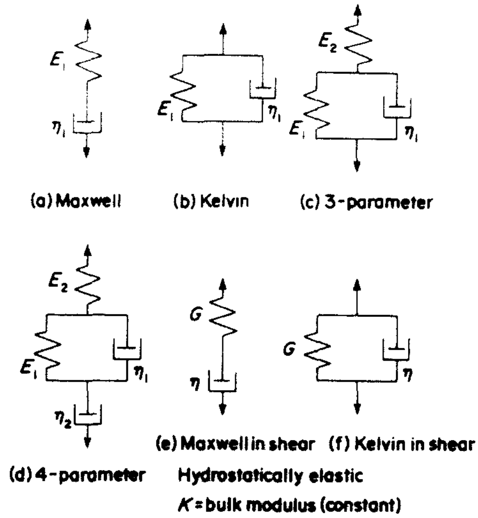

Fig. 3. Various viscoelastic models.

5. A PERTURBATION APPROACH

So far we have been concerned with the direct application of Berger's technique to the analysis of the behaviour of plates at elevated temperatures. However, as with all approximate methods there will be instances when this method does not yield sufficiently accurate solutions. In this case we may proceed using the perturbation technique presented in Refs. [19, 20] with a perturbation parameter of μ.

The governing equations for the classical analysis of thermally heated plates are [10]

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{33}$$

$$\frac{\partial N_y}{\partial y} \frac{\partial N_{xy}}{\partial y} = 0 \tag{34}$$

and

$$D\nabla^4 w - N_x \frac{\partial^2 w}{\partial x^2} - 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - N_y \frac{\partial^2 w}{\partial y^2} = q - \nabla^2 \frac{M_T}{1-\nu} - \rho h \frac{\partial^2 w}{\partial t^2} \tag{35}$$

where

$$N_x = hK(e - \mu\epsilon_y) - N_T/1 - \nu \tag{36}$$

$$N_y = hK(e - \mu\epsilon_x) - N_T/1 - \nu \tag{37}$$

$$N_{xy} = 0.5\mu Kh\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{38}$$

and where $K = E/1 - \nu^2$ is the plane stress stiffness, $e = (\partial u/\partial x) + (\partial v/\partial y)$ and $\mu = 1 - \nu$.

Let us now seek a solution of these equations in the form

$$u = \sum_{n=0}^{\infty} \mu^n u_n(x,y),$$

$$v = \sum_{n=0}^{\infty} \mu^n v_n(x,y),$$

$$w = \sum_{n=0}^{\infty} \mu^n w_n(x,y). \tag{39}$$

Since ν is always greater than zero the value of μ is less than unity so that we are in effect seeking a perturbation solution using a parameter of μ. Substituting eqn (39) into eqns (33)–(35) and equating coefficients of equal powers of μ we obtain the following system of equations for w_n, u_n and v_n, viz. for $n = 0$

$$D\nabla^4 w_0 + \left(\frac{N_T}{1-\nu} - khe_0\right)\nabla^2 w_0 = q - \nabla^2 \frac{M_T}{1-\nu} - \rho h \frac{\partial^2 w}{\partial t^2} \quad (40)$$

$$\frac{\partial}{\partial x}\left[khe_0 - \frac{N_T}{1-\nu}\right] = 0 \quad (41)$$

$$\frac{\partial}{\partial y}\left[khe_0 - \frac{N_T}{1-\nu}\right] = 0 \quad (42)$$

for $n \neq 0$

$$D\nabla^4 w_n - khe_n \nabla^2 w_n + kh\left[\epsilon_{n-1x}\frac{\partial^2 w}{\partial x^2} + \epsilon_{n-1y}\frac{\partial^2 w}{\partial y^2} + \gamma_{n-1}\frac{\partial^2 w}{\partial x \partial y}\right] = -\rho h \frac{\partial^2 w_n}{\partial t^2} \quad (43)$$

$$\frac{\partial e_n}{\partial x} - \frac{\partial}{\partial x}\epsilon_{n-1x} + \frac{1}{2}\frac{\partial}{\partial y}\gamma_{n-1} = 0 \quad (44)$$

$$\frac{\partial e_n}{\partial y} - \frac{\partial}{\partial y}\epsilon_{n-1y} + \frac{1}{2}\frac{\partial}{\partial x}\gamma_{n-1} = 0. \quad (45)$$

Here we have denoted

$$\epsilon_{nx} = \frac{\partial u_n}{\partial x}$$

$$\epsilon_{ny} = \frac{\partial v_n}{\partial y}$$

$$\gamma_n = \frac{\partial u_n}{\partial y} + \frac{\partial v_n}{\partial x}$$

$$e_n = \epsilon_{nx} + \epsilon_{ny}. \quad (46)$$

In the case $n = 0$ eqns (41) and (42) are readily integrated giving

$$khe_0 - \frac{N_T}{1-\nu} = f(t) \quad (47)$$

where $f(t)$ is an arbitrary function of time. When the edges of the plates are restrained from horizontal motion the value of the function $f(t)$ is given by $N_T^*/1-\nu$ so that the governing equation for w_0 reduces to

$$D\nabla^4 w_0 + \frac{E\alpha T^*}{1-\nu} h\nabla^2 w_0 = q - \nabla^2 \frac{M_T}{1-\nu} - \rho h \frac{\partial^2 w_0}{\partial t^2}. \quad (48)$$

This equation is the same equation as derived using Berger's hypothesis. Consequently we conclude that the solution of Berger's equations, for the response of thermally heated plates, represents the first term in a perturbation solution and that if more accurate solution is required then it is necessary to consider more terms in the series expansion given in eqn (39).

6. CONCLUSION

Thus we see that the Berger technique has been shown here to be a convenient method for obtaining close approximate solutions to the static and dynamic behaviour of heated plates. In fact, the work should also apply to cooled plates. The approach may be interpreted both as an extension of Williams method[8], for the analysis of long narrow plates, and also as a perturbation technique. Unlike existing techniques this method is simple to apply and makes use of existing results and routines for plate buckling and vibration. Recently the accuracy of the method has also been studied by the authors for large amplitude vibration of plates and membranes[21].

Acknowledgement—The authors wish to acknowledge the support of the Australian Research Grant Committee who sponsored this investigation vide Grant No. F75/15562.

REFERENCES

1. M. C. Pal, Large deflections of heated circular plates. *Acta Mech.* **8**, 82–103 (1969).
2. M. C. Pal, Large amplitude free vibration of circular plates due to aerodynamic heating. *Int. J. Solids Structures* **6**, 301–304 (1970).
3. M. C. Pal, Static and dynamic behaviour of heated orthotropic plates. *Int. J. Non-Linear Mech.* **8**, 489–504 (1973).
4. H. M. Berger, A new approach to the analysis of large deflections of plates. *J. Appl. Mech.* **22**, 465 (1955).
5. S. Timoshenko and J. M. Gere, *Theory of Elastic Stability*, 2nd Edn. McGraw-Hill, New York (1961).
6. M. Forray and M. Newman, On the post buckling behaviour of rectangular plates. *J. Aerospace Sci.* **29**, 754–755 (1962).
7. M. Newman and M. Forray, Axisymmetric large deflections of circular plates subjected to thermal and mechanical loads. *J. Aerospace Sci.* **29**, 1060 (1962).
8. M. L. Williams, Large deflection analysis for a plate strip subject to normal pressure and heating. *J. Appl. Mech.* **77**, 458–464 (1955).
9. P. Biswas, Thermal buckling of orthotropic plates. *J. Appl. Mech.* **21**, 361–363 (1976).
10. B. A. Boley and J. H. Weiner, *Theory of Thermal Stresses*. Wiley, New York (1960).
11. H. Lurie, Lateral vibrations as related to structural stability. *J. Appl. Mech.* **19**, 195–204 (1952).
12. R. Jones and J. Mazumdar, *Proc. Noise Shock and Vibration Conf.*, pp. 107–112. Transverse vibration of elliptic plates with inplane forces, Melbourne, Australia (1974).
13. A. W. Leissa, Vibration of plates. *NASA SP*-160 (1969).
14. C. D. Bailey, Vibration of thermally stressed plates with various boundary conditions. *AIAA* **121**, 14–19 (1973).
15. T. H. Lin, Creep deflection of viscoelastic plates under uniform edge compression. *J. Aeronautical Sci.* **23**, 883–887 (1956).
16. S. L. Deleeuw and G. E. Mase, Behaviour of viscoelastic plates under the action of inplane forces. *Proc. 4th U.S. Nat. Cong. Appl. Mech.* 999–1005 (1962).
17. S. L. Deleeuw, Circular viscoelastic plates subjected to inplane loads. *AIAA* **9**, 931–937 (1971).
18. S. L. Deleeuw, Buckling criterion for linear viscoelastic columns. *AIAA* **11**, 2665–2666 (1963).
19. R. Schmidt, On Berger's method in the non-linear theory of plates. *J. Appl. Mech.* **41**, 521 (1974).
20. R. Schmidt and D. DaDeppo, A new approach to the analysis of shells plates and membranes with finite deflections. *Int. J. Non-Linear Mech.* 409–419 (1974).
21. J. Mazumdar and R. Jones, A simplified approach to the large amplitude vibration of plates and membranes. *J. Sound and Vibration* **50**, 389–397 (1977).